

(iv)  $X = \{(x_1, x_2) \mid x_2 - 3 \geq -x_1^2, x_1 \geq 0, x_2 \geq 0\}$  is not a convex set

$u = (0, 3) \in X$

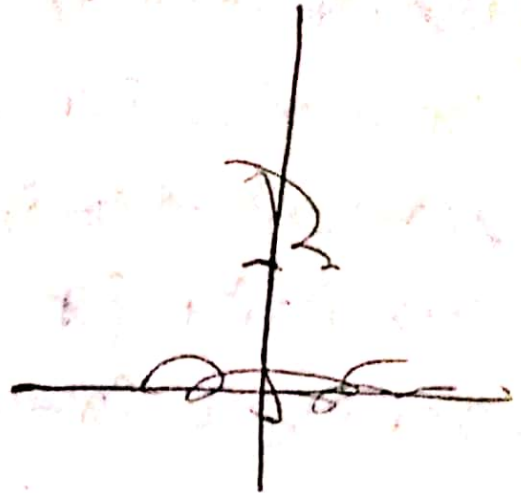
as  $0 \geq 0, 3 \geq 0$

and  $3 - 3 = 0 \geq -0^2$

and  $v = (2, 0)$

$v = (2, 0) \in X$  as  $2 \geq 0, 0 \geq 0$

$0 - 3 = -3 \geq -2^2 = -4$



If we choose  $\lambda = 0.5$

Then their convex combination.

$$w = \lambda u + (1-\lambda)v$$

$$= 0.5(0, 3) + 0.5(2, 0)$$

$$= (0, 1.5) + (1, 0) = (1, 1.5)$$

Now  $1.5 - 3 = -1.5 < -1$

So  $w \notin X$

Hence  $X$  is not a convex set.

Ex.  $X = \{(x_1, x_2) \mid x_1, x_2 \leq 1, x_1, x_2 \geq 0\}$  is  
not a convex set

We see that  $u = (0.6, 0.4) \in X$

as.  ~~$0.6 \leq 1, 0.4 \leq 1$~~   $0.6 < 1, 0.4 < 1$

$$0.6 \times 0.4 = 0.24 > 0$$

and  $v = (-0.5, -0.5) \in X$  as.

$$-0.5 < 1 \text{ and } (-0.5)(-0.5) = 0.25 > 0$$

But if we choose  $\lambda = 0.5$

then  ~~$\lambda u + (1-\lambda)v$~~  which then their  
convex combination is

$$w = \lambda u + (1-\lambda)v = 0.5u + 0.5v$$

$$= 0.5(u+v)$$

$$= 0.5((0.6, 0.4) + (-0.5, -0.5))$$

$$= 0.5(0.1, -0.1)$$

$$= (0.05, -0.05)$$

Now  ~~$0.05 < 1, -0.05 < 1$~~   $0.05 < 1, -0.05 < 1$

$$\text{but } -0.05 \times 0.05 = -0.0025 < 0$$

So  $w \notin X$

Hence  $X$  is not a convex set.